The Depth of Sunlight Penetration in Cloud Fields for Remote Sensing

A. A. Kokhanovsky

***Abstract*—This letter is devoted to the derivation of a simple approximate equation for the depth of sunlight penetration in a cloud field. This depth is defined as the cloud optical thickness, which corresponds to the reflection function equal to 90% of the reflection function for a semi-infinite cloudy layer.**

***Index Terms*—Clouds, radiative transfer, remote sensing.**

I. Introduction

The PENETRATION depth is a parameter required in various remote sensing applications. It is defined as the length at which the intensity of incident wave is reduced by the factor $\exp(-1)$. If the absorption coefficient of a homogeneous medium under study is known, the penetration depth can be easily estimated. It could be of value to extend this notion to random media. However, this leads to a number of problems. In particular, let us take a cloud in the sky. In this case, a strongly developed multiple light scattering occurs in the medium. The downward diffused light intensity reaches a maximum and starts to decrease, preserving the angular pattern of scattered light. Brightness of this pattern decreases as $\exp(-k\tau)$, where k is the diffusion exponent, and τ is the optical depth.

Space remote sensing applications require information on the thickness of an effective layer of a given random medium that interacts with incident electromagnetic radiation. This depth can be defined as the distance ℓ at which the reflection function reaches 90% of its value for the semi-infinite layer. We can state that scattering layers positioned at depths larger than ℓ only weakly influence the signal detected by an orbiting optical instrument.

The task of this letter is to present simple analytical equations, which can be used for estimations of ℓ in a cloudy atmosphere. Generally, the equations derived can be used inside and outside gaseous absorption bands. However, in the results of the numerical calculations presented, we neglect the influence of the gaseous absorption bands on the value of the penetration depth.

II. Theory

Generally speaking, the penetration depth can be found using results of the radiative transfer calculations similar to those presented in Fig. 1, where we show the dependence of the reflection function [1] on the cloud optical thickness for multiple wavelengths. Data for Fig. 1 were obtained using the discrete ordinate method of the radiative transfer equation solution. It was assumed that droplets are characterized by the gamma particle size distribution (PSD) with the effective radius of 6 μm and the coefficient of variance of the PSD equal to 38% [2]. Calculations were performed at $\xi = 0.5, \eta = 1.0$ (nadir observation). Here, ξ is the cosine of the solar angle, and η is the cosine of the observation angle. Obviously, the results do not depend on the relative azimuth φ for the nadir observation conditions.

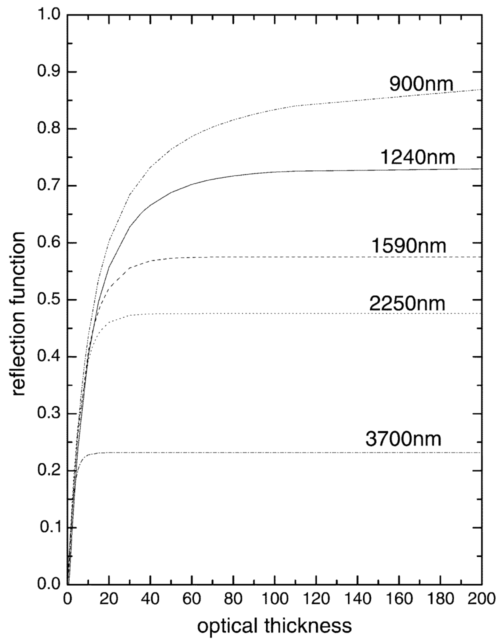

Fig. 1. Dependence of the reflection function on the optical thickness for several wavelengths of incident light (see details in text).

It follows from Fig. 1 that the penetration depth decreases with wavelength. This is mostly due to the fact that water absorption generally increases with the wavelength [2]. In particular, we find from Fig. 1 that the optical penetration depth $\tau_p = \ell/s$ (with s as the photon-free path in a cloud) is equal to 65.5, 37.0, 20.0, 13.2, and 5.7 for wavelengths $\lambda = 900$, 1240, 1590, 2250, and 3700 nm, respectively.

Computations as those shown in Fig. 1 require a radiative transfer code. Let us show that the penetration depth can also be found using a simple analytical equation. For this, we will use the results given in [1] and [2]. In particular, the reflection function $R(\xi, \eta, \varphi, \tau)$ can be presented as

$$R(\xi,\eta,\varphi,\tau) = R_\infty(\xi,\eta,\varphi) - t(\tau)\exp(-x-y)K_0(\xi)K_0(\eta) \quad (1)$$

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The author is with the Institute of Environmental Physics, University of Bremen, 28334 Bremen, Germany, and also with the Institute of Physics, Minsk 220072, Belarus (e-mail: alexk@iup.physik.uni-bremen.de).

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where $R_\infty(\xi,\eta,\varphi)$ is the reflection function of a semi-infinite medium having the same microphysical characteristics as a finite layer under study. The function $t(\tau)$ is the global transmittance given approximately by

$$t(\tau) = \frac{\sinh(y)}{\sinh(x+\alpha y)}. \tag{2}$$

Here, $x = k\tau, y = 4k/[3(1-g)], k = \sqrt{3(1-g)(1-\omega_0)}, \omega_0 = 1 - \sigma_{\text{abs}}/\sigma_{\text{ext}}, \alpha \approx 15/14, g$ is the asymmetry parameter of the phase function, σ_{abs} is the absorption coefficient, and $\sigma_{\text{ext}} \equiv 1/s$ is the extinction coefficient. The accuracy of (1) has been thoroughly studied in [3], [4]. It was found that (1) can be applied with an accuracy better than 5% for a cloud optical thickness larger than 10 and a wavelength smaller than 1500 nm for most observation geometries. The accuracy increases with the optical thickness and the single-scattering albedo. It can also be applied for larger wavelengths providing that the single-scattering albedo is larger than approximately 0.95. This is the case for clouds in visible and most of the near-infrared [2].

The escape function $K_0(\xi)$ in (1) can be approximated by the following simple equation at $\xi > 0.2$ [2], [3]:

$$K_0(\xi) = \frac{3}{7}[1+2\xi]. \tag{3}$$

The accuracy of this equation is better than 2% at $\xi \geq 0.2$ [3].

It follows from (1) at $\tau = \tau_p$ that

$$ut(\tau_p)\exp(-x(\tau_p) - y(1-u^*)) = b \tag{4}$$

where $b = 0.1$. We used the following approximate result for the reflection function of a semi-infinite weakly absorbing medium [2]:

$$R_\infty(\xi,\eta,\varphi) = R_{\infty 0}(\xi,\eta,\varphi)\exp(-u^* y) \tag{5}$$

where $R_{\infty 0}(\xi,\eta,\varphi)$ is the reflection function of a semi-infinite medium, under the assumption that absorption of radiation in a cloud does not take place and

$$u^* = (1-0.05y)u, \tag{6}$$

$$u = \frac{K_0(\xi)K_0(\eta)}{R_{\infty 0}(\xi,\eta,\varphi,\tau)}. \tag{7}$$

Equation (5) is accurate to within 5% at $y \leq 1.7$ [5]. After simple algebraic derivations, it follows from (4) that

$$\tau_p = \frac{1}{2k}\ln\{2pu\sinh(y)\exp(-(1-u^*)y) + \exp(-\alpha y)\} - \frac{2\alpha}{3(1-g)} \tag{8}$$

where $p = 1/b$. We have from (8) at $k = 0$ after taking a limit

$$\tau_p = \frac{4(pu-\alpha)}{3(1-g)}. \tag{9}$$

Equation (8) gives the result we intended to gain from the very beginning. The dependence $\tau_p(\lambda)$ for a water cloud having the same microphysical characteristics and observation conditions as those used in Fig. 1 are shown in Fig. 2. The Mie theory was used to calculate $\omega_0, \sigma_{\text{ext}} = 1/s$, and g in (8). The gaseous absorption was neglected. Symbols correspond to values of τ_p obtained from the data given in Fig. 1. We see that our approximate equation can be used for an accurate estimation of the sunlight penetration in clouds. Note that values obtained for $\tau < 10$ may be biased, as the accuracy of (1) decreases in these cases. However, we do not restrict this plot to values below 2 μm to show the general trend of $\tau_p(\lambda)$. Furthermore, the point at 3.7 μm indicates that (8) might even be used at $\tau_p \in [5, 10]$. The following approximate result for the function $R_{\infty 0}(1,\xi)$ was used in the calculations presented in Fig. 2 [6]:

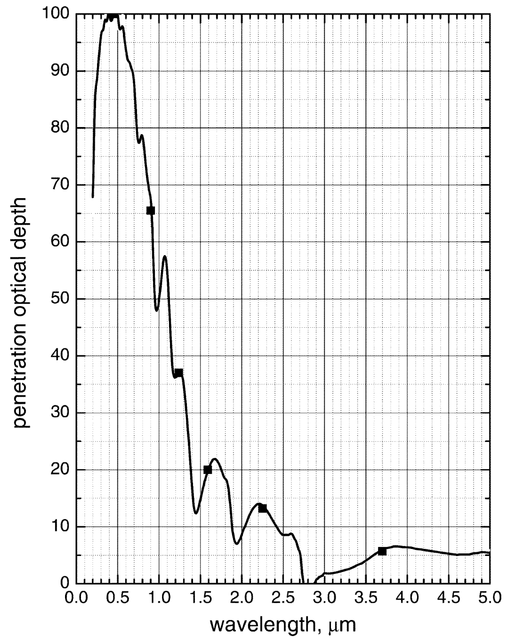

Fig. 2. Dependence of the penetration optical thickness on the wavelength obtained using (8) for water droplets having the effective radius 6 μm at the nadir observation and the solar angle equal to 60°. Symbols give the results obtained using the numerical solution of the integro-differential radiative transfer equation.

$$R_{\infty 0}(1,\xi) = \frac{0.37 + 1.94\xi}{1+\xi}. \tag{10}$$

This equation is accurate to within 5% for the nadir observation and most of the solar incident angles used for the cloud remote sensing (oblique incident angles are excluded). The useful approximate equation for the function $R_{\infty 0}(\xi,\eta,\varphi)$ at nonnadir observation conditions is given in [7].

Fig. 2 quantifies the result already mentioned above. Namely, the optical penetration depth generally decreases with the wavelength. However, at some narrow spectral intervals, the opposite is true (e.g., see the region close to 1 μm in Fig. 2). The value of τ_p changes from approximately 100 at $\lambda = 0.5$ μm to approximately 6 at $\lambda = 3.7$ μm for the typical case shown in Fig. 2. Using a typical value of $\sigma_{\text{ext}} = 50$ km^{-1}, we obtain that the ℓ changes from 120 m at $\lambda = 3.7$ μm to 2000 m at $\lambda = 0.5$ μm. In fact, Fig. 2 is also applicable to ℓ after the rescaling of the ordinate, e.g., the ordinate should be multiplied by 10 at $\sigma_{\text{ext}} = 100$ km^{-1}. Then, ℓ is given in meters.

It follows from Fig. 2 that the radiance detected at different wavelengths originates in part from different cloud depths.

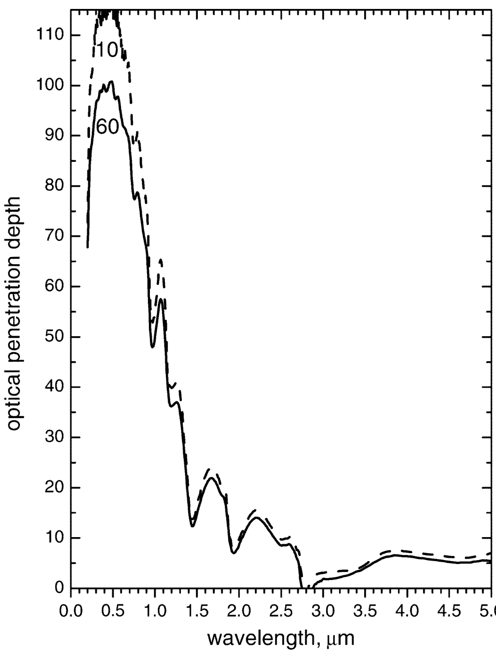

Fig. 3. Same as in Fig. 2, except that the results for the solar angle 10° are also shown.

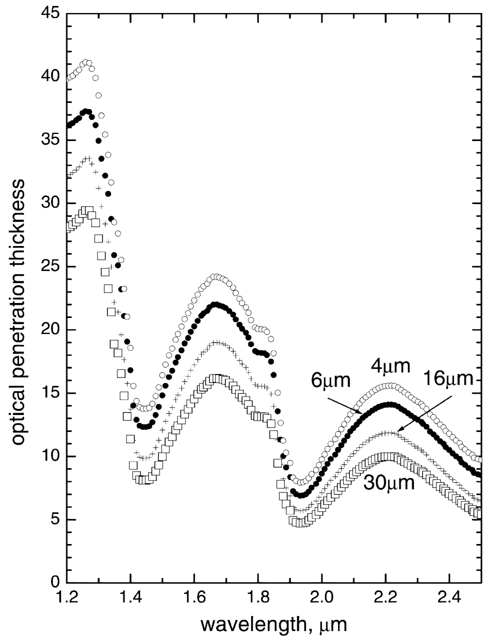

Fig. 4. Same as in Fig. 2, except that the results for the several values of the effective droplet radius are shown.

This sets an important question as far as cloud satellite remote sensing is concerned. Namely, the cloud liquid water path (LWP) and the effective radius of droplets are usually obtained from measurements of reflectances at multiple wavelengths. This leads to no complications for homogeneous clouds. However, homogeneous clouds do not exist, e.g., the size of droplets usually increases from the bottom to the top of a cloud layer. It means that the radius of droplets obtained at 3.7 μm [$a_{\mathrm{ef}}(3.7\ \mu\mathrm{m})$] is not necessarily representative for a whole cloud. In this case, the derivation of the liquid water path (LWP) as the product of the optical thickness in visible and $a_{\mathrm{ef}}(3.7\ \mu\mathrm{m})$ (we omit a numerical multiplier [2]) may bias the LWP derived considerably. Therefore, it is of importance to specify the wavelengths used to derive a_{ef} and the LWP, while referring to their values obtained from the optical instruments onboard satellites. Generally, decreasing the wavelength will lead to smaller values of a_{ef} derived (and also smaller values of the LWP).

It follows from Fig. 3 that the value of τ_p is larger for the illumination closer to the normal. This can easily be explained by the fact that photons injected into the medium along oblique angles escape more quickly and do not penetrate as deeply as photons incident on a given medium along the normal.

The spectral dependence of the optical penetration thickness is shown in Fig. 4 for various sizes of particles. We see that the value of τ_p decreases with the size of particles in the infrared. This can be expected from the larger light absorption by larger droplets. We also found that the optical penetration depth is slightly larger for larger droplets in the visible. This is due to larger values of g for clouds with larger droplets. The account for the gaseous absorption will modify the data shown in Fig. 4, adding an oscillating part on the general background curve depending on the gas type/concentration. However, we do not consider this contribution in any detail here.

Equation (8) becomes less accurate for values of the single-scattering albedo smaller than 0.95. This case may be of importance for polluted clouds and also for ice clouds having large crystals and, therefore, increased value of light absorption. In this case, the problem can be solved using the general asymptotic equation valid for cloud optical thicknesses larger than 10 and an arbitrary single-scattering albedo. This equation has the following form [8]:

$$R(\xi,\eta,\varphi,\tau) = R_\infty(\xi,\eta,\varphi)[1 - ml\bar{u}(\xi,\eta,\varphi,\tau) \times (1 - l^2\exp(-z))^{-1}\exp(-z)] \quad (11)$$

where $\bar{u}(\xi,\eta,\varphi) = K(\xi)K(\eta)/R_\infty(\xi,\eta,\varphi), z = 2k\tau$, and therefore

$$\tau_p = (2k)^{-1}\ln(ap + l^2) \quad (12)$$

where

$$a = mlK(\xi)K(\eta)/R_\infty(\xi,\eta,\varphi). \quad (13)$$

The issues related to the numerical calculation of asymptotic parameters and functions $k, l, m, K(\xi)$, and $R_\infty(\xi,\eta,\varphi)$ are discussed in [8]–[10]. However, note that the essential simplicity characteristic to (8) is lost then.

Equation (1) follows from (11), assuming that

$$l = \exp(-\alpha y) \quad mK(\xi)K(\eta) = (1-\exp(-2y))K_0(\xi)K_0(\eta) \quad (14)$$

as shown in [3]. The same correspondence exists between (8) and (12) [with account for (5)].

III. Conclusion

A simple expression for the depth of sunlight penetration in the cloud field is derived. It can be used for rapid estimations of cloud depths, which interact with sunlight at a given

wavelength, providing that the cloud optical thickness is larger than 10. The single-scattering albedo is assumed to be larger than approximately 0.95, which is the case for most of the visible and near-infrared regions. The derivation is based on the modified asymptotic theory valid for optically thick weakly absorbing clouds, which are characterized by comparatively large values of the reflection function [1]. It means that the results for highly absorbing wavelengths (e.g., see the curve in the vicinity of 3 μm in Fig. 2) may be biased. The comparison of the calculations using (8) and the exact radiative theory confirms a high accuracy of the expression developed (see Fig. 2). Equation (8) is not limited to a cloud medium only, but can be applied to other disperse media including snow, ice, and leaves.

Although the gaseous absorption was neglected in the results of the calculations given in this letter, it could easily be taken into account if one modifies the single-scattering albedo to account for an additional absorber. Furthermore, in this case, the value of p in (8) needs to be multiplied by the product of atmospheric transmittances from the top of the atmosphere to the cloud and from the cloud to a satellite, as discussed in [11].

It is stated that, despite that multispectral measurements can give us information on several characteristics of clouds (as compared to just one characteristic for a single-wave sensing), special attention should be paid to the interpretation of measurements. In particular, measurements at several wavelengths may refer to different depths of a cloudy medium. The same applies to different illumination/observation conditions (see Fig. 3). This is solely due to the inhomogeneity of cloud local optical characteristics (for example, the vertical structure of light absorption and extinction caused by vertical profiles of liquid water content and the size of droplets, e.g., see [12]–[14]).

Not entirely the same cloud depths are studied using multiple wavelengths. Therefore, retrieved characteristics represent not only *in situ* cloud properties but also the spectral intervals used for remote sensing purposes. These intervals should also be specified together with the characteristics retrieved. On the other hand, remote sensing at several wavelengths may reveal the vertical structure of the medium under study.

We stress that the penetration depth depends on the single-scattering albedo. So it is reduced for polluted clouds in the visible depending on the level of pollution. It also decreases with the size of particles and the incident angle (for nadir observations). Generally, optically thick crystalline clouds are characterized by smaller penetration depths as compared to warm clouds. This is mostly due to the large size of ice particles and their irregular shape.

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